Analysis of Thin Shell Structures

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Introduction

In this paper, I will investigate thin-shell concrete structures. First, I will describe general characteristics of the structure with a focus on how the amazing spans are achieved. Second, I will use three different examples to further explain the possibilities of the structure, focusing on distinct features of each. Third, I will briefly discuss the application and development of thin-shell concrete structures in the future.

There are two reasons why it would be interesting to research thin-shell concrete structures. To begin with, they are structurally efficient and economically suitable and thus play a substantial role in history. Thin-shell structures stemmed from the Modern Movement in architecture which created an aesthetic impulse for buildings with dramatically long spans unencumbered by columns (Thomas E. Boothby, 2012). In addition, the Modern Movement in architecture idealized industrial architecture as expressive of the application of modern forms and materials; it celebrated structural technology by exposing the materials and frameworks of buildings (Thomas E. Boothby, 2012). Under such a circumstance, the thin-shell structures were widely built throughout the entire world and became the International Style during the period from 1920 to 1970 (Peerdeman, 2008). Furthermore, future revival of part of these structures is likely. Contemporary architecture seems to experiment more with undefined free-form structures (Peerdeman, 2008); one example is the EPFL Learning Center by SANAA (Peerdeman, 2008). Historic thin-shell structures would be important examples to be consulted.

Thin-Shell Structure

A thin shell structure is a structure composed of a curved slab whose thickness is small compared to its other dimensions and compared with its principal radius of curvature (Buyukozturk, 2004). Thin shells cover medium to large spans; a thin shell has a very small thickness-to-minimal-radius ratio, often smaller than 1/50 (J. Blaauwendraad, 2014).

The major reason why shells are such strong and economic structures is that shells can carry out-of-plane loads by in-plane membrane forces (J. Blaauwendraad, 2014). I will next better explain this by comparing shell with plates. Both shells and plates are structures of which the dimensions in two in-plane directions are large compared to the third direction perpendicular
to the plate. In other words, both cover a large span but has a comparatively negligible thickness. Consequently, shells and plates are similar in that they are both defined by their middle plane, thickness and material properties (J. Blaauwendraad, 2014), where the middle plane refers to the plane parallel to surfaces of the plate or the shell that divides the thickness of the plane or the shell into two halves (Eduard Ventsel, 2001), as is illustrated by Fig. 1 and Fig. 2. They are different in that the middle plane of plates is flat whereas the middle plane in shells is curved (J. Blaauwendraad, 2014). As a result, as for plates, in-plane loads generate in-plane membrane forces, and out-of-plane loads generate moments and transverse shear forces (J. Blaauwendraad, 2014)). In contrast, as for shells, both out-of-plane and in-plane loads can be carried in shells by in-plane membrane forces, which is not possible for plates (J. Blaauwendraad, 2014). This is further illustrated by Fig. 3 in which the α- and β-coordinate lines are mutually perpendicular at all points on a surface Ω (Eduard Ventsel, 2001). And such loads are uniformly distributed over the shell thickness.

Fig. 1 Middle Plane of Plates

Fig. 2 Middle Plane of Shells

Fig. 3 In-plane Membrane Force
There are two theories that explain the behavior of thin shell structures. The first theory is called membrane theory. Membrane theory deals with the undisturbed and major part of the shell which behaves like a true membrane, and such a state of stress is called membrane state of stress. The main characteristic of this state of stress is that arbitrary external loading can be supported by means of stress resultants (internal forces) and stress couples (bending and twisting moments) (Eduard Ventsel, 2001). However, stress couples (bending and twisting moments) are either zero, or so small that they may be neglected (Eduard Ventsel, 2001). This unique property of shells is a result of the curvature of the spatial structure (J. Blaauwendraad, 2014). Owing to the shell curvature, the projections of the direct forces on the normal to the middle surface develop an
analog of an ‘elastic foundation’ under the shell (Eduard Ventsel, 2001). So, it can be said that a shell resists an applied transverse loading as a flat plate resisting on an elastic foundation. This phenomenon is explained in Fig. 4.

However, membrane theory is not sufficient to explain the behavior of thin shell structures in all cases. For example, in Fig. 5, membrane theory is weak in expelling the boundary conditions and deformation constraints, the case of concentrated loads, and the case where the geometry of a shell is not perfectly continuous but suddenly changes at a line. Therefore, the other theory is introduced which is called bending theory. Bending theory deals with bending that is conned to boundaries; this is called edge disturbance in contrast to the undisturbed membrane (J. Blaauwendraad, 2014). However, bending theory will not be discussed in detail in this paper.

Conventional concrete mixture and high strength fibre reinforced concrete are two potential materials for thin shell structures. Conventional concrete mixture refers to that composed mainly of water, aggregate, and cement. For high strength fibre reinforced concrete, silica fume, steel fibers, super plasticizers are added with different proportions; the concrete can be further reinforced if fibers with different dimensions are added (Peerdeman, 2008). Thin shell structure should be distinguished from arches in terms of the distribution of stresses; while an arch of a given form will support only one completely determined load without bending, a shell of a given shape has, provided its edges are suitable supported, as a rule, the same property of a wide range of loads which satisfy only very general requirements (Eduard Ventsel, 2001). And both compression and tension may exist within a thin shell structure. The structural behavior as such cannot be realized without materials that have complicated nonlinear behaviors (Peerdeman, 2008). Both conventional concrete mixture and fibre reinforced concrete are feasible in this sense.

However, relatively, though conventional concrete shows high compressive strength, it cannot resist much of tensile stress. Thus, high strength fibre reinforced concrete is preferable. And there is empirical evidence that stresses in concrete shell constructed from (fibre) reinforced concrete ‘are often a small proportion of those permitted by the strength of the material; and that
‘the foot of the shell is the only point where the stresses go above 50% of the allowable concrete strength’ (Peerdeman, 2008).

I will not give detailed classifications of thin shell structure in this paper. I just show some examples of different types of thin shell structure based on different geometries Fig. 6. Among these, the most basic forms of thin shell structure are paraboloid of revolution and hyperbolic paraboloid.

Lastly, I will discuss briefly another two important points about thin shell structure. First, the main issue of thin-shell concrete structure is roofing. Shell geometries can be highly
complex. There might be excessively steep and excessively flat or shallow roof slopes. This makes it challenging to maintain a water tight roofing membrane and to address the potential for concrete deterioration (Thomas E. Boothby, 2012).

The Chapel of Lomas de Cuernavaca by Felix Candela

For the Chapel of Lomas de Cuernavaca, or Cuernavaca Chapel, Felix Candela employed a dramatically modified saddle shape hyperbolic paraboloid (Fig. 7, Fig. 8). The taller free edge the front soars to a height of 21.9 m; at its widest point, the shell spans 31 m (Fig. 9) (Powell Draper, 2008). The shell’s thickness is primarily 4 cm throughout, but it is thickened along the sides (Powell Draper, 2008).

The structure is efficient in that bending is minimized. This is realized thanks to the hyperbolic paraboloid form of the structure. However, in usual cases, when large moments arise in a shell structure, greater thickness is required to assume and distribute the load without excessive deformation, cracking, or failure (Powell Draper, 2008). But this obscures the structure’s display of its thinness, which Candela valued a lot (Powell Draper, 2008). The analysis done by Powell Draper provides empirical evidence for the structural efficiency of
shells. According to Draper (2008), principal shell stresses reach a maximum of 4,488 kPa in compression and 1,131 kPa in tension. Both are well under the load-bearing limit of concrete. He also mentions that maximum deformation in their finite-element analysis model was 0.5 cm; no significant cracking was observed in the actual structure so they conclude that deformations are kept within reasonable limits (Powell Draper, 2008).

One important aspect that contributes to the structural performance of shells is the thickened edge. ‘At the chapel sides where the structure connects to the foundation, the shell forces are concentrated, so Candela thickened it sufficiently to reduce the stress in the
structure’ ("MAJOR WORKS, CHAPEL LOMAS DE CUERNAVACA," 2008). Fig. 10 and Fig. 11 show how this is done in the actual structure. According to Draper (2008), edge beam with varying thickness and an edge beam, as the actual structure has, is the most efficient structure compared shells of uniform thickness and shells of uniform thickness but with edge beam. The finite-element model results for the comparison is shown in Table 1. It is clear from the table that a structure with varying thickness shows a general reduction of stress from the models with uniform thickness and a more gradual distribution of stress throughout the shell (Powell Draper, 2008). Meaning, bending is better minimized.

<table>
<thead>
<tr>
<th></th>
<th>Uniform thickness</th>
<th>Uniform thickness with edge beam</th>
<th>Varying thickness with edge beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max displ. [cm (in.)]</td>
<td>-0.5 (~0.2)</td>
<td>-0.5 (~0.2)</td>
<td>-0.3 (~0.1)</td>
</tr>
<tr>
<td>Min principal stress [kPa (psi)]</td>
<td>-4,492 (~651)</td>
<td>-4,693 (~681)</td>
<td>-2,525 (~366)</td>
</tr>
<tr>
<td>Max principal stress [kPa (psi)]</td>
<td>1,134 (164)</td>
<td>1,586 (230)</td>
<td>1,505 (218)</td>
</tr>
</tbody>
</table>

Another important merit of Felix Candela’s Chapel of lomas de Cuernavaca is the connection between its form and its function. It is clear from previous analysis that nothing was done for purely aesthetic effect; the choice of form was inextricably connected to its structural purpose ("MAJOR WORKS, CHAPEL LOMAS DE CUERNAVACA," 2008). However, this basic structural form of the hyperbolic paraboloid does not bind the designer into a predetermined geometry (Powell Draper, 2008). Within the discipline of the hyperbolic paraboloid, Candela found room for creative design, by maximizing the effect of the free edge with a strikingly tall opening. It is true that such modification to the basic structural form creates the issue of increased dead load and wind load; Candela countered by adeptly thickening the
edges and back of the structure. The result is that the structure adheres to an efficient geometrical form, but it can also be viewed as a piece of art.

**The TWA Flight Center at New York’s JFK Airport by Eero Saarinen**

The exterior of the TWA terminal is composed of remarkably few elements, and its simplicity is furthered by the two building materials: concrete buttresses and green-tinted glass walls (Commission, 1994). The wing-like roof of the central portion rises above low wings that extend on the east and west, and follow the curve of the airport service road. Extending from the main terminal are two raised walkways that connect with gate structures on the aircraft ramp; the two-story eastern gate structure has a pair of remote gate lounges. The exterior concrete area of terminal are painted in a range of cream shades. Four complexly-massed piers support the roof over the central portion of the terminal. The four segments of the roof, separated by narrow skylights, meet at the central roof plate. They enclose the terminal and are described as four lobes or segmental domes, each of which stands alone, resting on two buttress supports as is illustrated in Fig. 12 and Fig. 13 (Commission, 1994). They are also outwardly-canting, extending as far as eighty feet. Fig. 14, the construction of the TWA Flight Center better shows how the roof rests on the buttress supports.
Fig. 13 TWA Flight Center Structure 2

Fig. 14 Construction of TWA Flight Center
I will start by giving general descriptions of the cast-in-place folded thin shell reinforced concrete roof. The roof consists of eight thin shell structural units. Each unit comprises of four hyperbolic paraboloid shells monolithically joined together along a centerline to form a V-shaped cross-section, as is illustrated in Fig. 16 (Sigrid Adriaenssens, 2014). The V-shaped unit is supported by three inclined columns—two at the back and one at the interior. The unit cantilevers 20.2 m forward from the interior column over the stands below towards the water (Sigrid Adriaenssens, 2014). The folds are joined together via a keyed joint filled with concrete grout which also contain steel weld tabs that prevent relative translation between adjacent folds (Sigrid Adriaenssens, 2014). The Miami Marine Stadium is unique in that no other realized concrete structure has a series of cantilevering hypar units monolithically attached and linearly placed next to each other, and such a system of attached units leads to better structural performance (smaller bending moments and increased stiffness) compared to a system of individual units (Sigrid Adriaenssens, 2014). The roof structure region between the back and interior columns is referred to as the backspan and the overhang is referred to as the cantilever as illustrated in Fig. 17. On top of the roof, the backspan and cantilever are separated by a concrete beam, referred to as the stiffener. The stiffener is located above the line that connects the tops of the interior columns.
I will next give specific analysis for the hyperbolic paraboloid shell roof. Hyperbolic parabolic shells can be visualized as two systems of arches, one downward curving parabola in compression and one upward curving parabola in tension (Fig. 16) (Sigrid Adriaenssens, 2014). Fig. 18 better illustrates the difference between two arches. The arch forces are brought to the straight line edges (or edge beams and groin folds in the case of the Miami Marine Stadium) where the components perpendicular to these edges cancel and the components parallel to the edges add to give shear forces along the edge beams and fold groins (Sigrid Adriaenssens, 2014). The edge beams and fold groins in turn carry the shear forces by axial tension or compression. In the Miami Marine Stadium shells the lower groin folds carry compression forces to the interior and back columns while the higher groin folds and exterior edge beams carry tension (Sigrid Adriaenssens, 2014).

What I find special about this building is Hilario Candela’s concern about the materiality of buildings. As discussed previously, concrete is one of the ideal materials for thin shell
structures, and in the Miami Marine Stadium, such a decision out of structural requirement and economic functionality also serve aesthetic purposes. According to Candela, who is in love with Dulles Airport at Washington, DC (Fig. 19), ‘[Saarinen] placed the roof on top like canopy. Every column is gorgeous. The way the form of those columns gets to the ground and human beings can touch the concrete and feel it and be next to it… This is exactly what I was after’ (Sigrid Adriaenssens, 2014). Hence, he further developed and maximized the ‘softness and strength’ of concrete in the Miami Marine Stadium.

Fig. 18 The Downward Curving Parabola and the Upward Curving Parabola

Fig. 19 The Interior of Dulles Airport at Washington, DC
Conclusion and Future Development

Given previous analysis of thin shell structures in general and analysis of Cuernavaca Chapel, TWA Flight Center, and Miami Marine Stadium, I will give a conclusion as follow. First, concrete shells are efficient structures and can be used as durable solutions for roofs or for covering large spaces. Second, concrete shells can be built with limited thicknesses, and necessity for interior columns is reduced, thus maximizing the space inside them (Aurelio Muttoni, 2013). However, for the thin shell structure to be widely applied in contemporary buildings, a more efficient construction technique has to be developed. According to the statistics of a concrete shell covering a mall at Chiasso, Switzerland, the cost of the concrete structure was: “49 % for falsework and formwork, 21 % for ordinary reinforcement, 5 % for post-tensioning, 24 % for sprayed concrete and 1 % for poured in situ concrete. (Aurelio Muttoni, 2013)” This reveals the relatively large cost of false work and formwork for this type of structure, and points to the need for future research on more efficient construction techniques.
Bibliography


Peerdeeman, B. (2008). Analysis of Thin Concrete Shells Revisited: Opportunities due to Innovations in Materials and Analysis Methods. (Master), Delft University of Technology.

